Bell tests with photon-entanglement:
LHV models and critical efficiencies at the light of Wigner-PDC optics

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Within the Wigner-PDC picture of photon entanglement, detection "errors" are not independent (though they may look, on average), nor can they be controlled by means of a technological improvement on the detectors. Those two elements make possible the interpretation of experimental evidence without the need to exclude local realism: for that reason, we propose the abandonment of the usual (photon, particle-based) description of (PDC-generated) light states, in favour of an also quantum, but field-theoretical description (QED), a description that finds a one-to-one equivalent in that Wigner-PDC approach.

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In recent posts we spoke of (detection probability) *enhancement* and detector inefficiencies as "natural features of the Wigner-PDC picture of photon entanglement". We are the first to admit that, left at that stage, such an assertion looked rather vague. Well, here we will try to give it some materiality, at least in relation to detector inefficiencies, we have talked about enhancement somewhere else: for instance see sidenotes of [7]. First of all, we recall that the Wigner-PDC picture [2] provides a formalism that corresponds, one-to-one (and therefore without leaving the orthodoxy of Quantum Electrodynamics at any stage, a fact not always sufficiently acknowledged), to a field theoretical formulation of photon entanglement, when generated using the well known (and nowadays almost standardized) technique of Parametric Down Conversion (PDC). In general we will refer to polarization-entanglement, but so far to my knowledge such entanglement can also be explained in the same way, always within the context of PDC, for other degree of freedom: time, path, energy... see [4].

Recently we have proven that such Wigner-PDC picture can be cast, by means of an elementary mathematical manipulation, into an entirely local realistic form, where single and double detection probabilities, respectively, acquire the expressions (following our general conventions in [7] but for minor details, for simplicity):

$$P_i^{(W)}(det) = \int_\alpha f_i(\alpha) \, W(\alpha) \, d\alpha, \qquad (1)$$

$$P_{i,j}^{(W)}(det) = \int_\alpha \Gamma_{i,j}(\alpha) \, W(\alpha) \, d\alpha, \qquad (2)$$

where α is a vector of hidden variables grouping all (relevant) vacuum mode amplitudes going into the crystal (the source), its distribution governed by $W(\alpha)$, the Wigner function of the vacuum [3], and perhaps (if other devices are interposed in the light rays path), also the

ones entering every other device in the setup, i.e.

$$\alpha \equiv \alpha_s \oplus \alpha_1 \oplus \ldots \oplus \alpha_N, \qquad (3)$$

where subindexes indicate the source (s) and the other N devices in the setup. All α's are defined there to be independent from each other, therefore their joint density function factorable. In consistency, we can interpret

$$P_i^{(W)}(det|\alpha) \equiv f_i(\alpha), \qquad (4)$$

$$P_{i,j}^{(W)}(det|\alpha) \equiv \Gamma_{i,j}(\alpha), \qquad (5)$$

where detector subindexes can be omitted if all of them are physically identical. The fact that $\Gamma_{i,j}(\alpha)$ is in general non-factorable [9], i.e., in general

$$\Gamma_{i,j}(\alpha) \neq f_i(\alpha) \cdot f_j(\alpha), \qquad (6)$$

can be interpreted, using some customary terminology within the field of Quantum Information, as "the errors are not independent", which does not mean that their overall integral with respect to α cannot well satisfy

$$P_{i,j}^{(W)}(det) = P_i^{(W)}(det) \cdot P_j^{(W)}(det), \qquad (7)$$

i.e.,

$$\int \Gamma_{i,j}(\alpha) \, W(\alpha) \, d\alpha =$$
$$\left[\int f_i(\alpha) \, W(\alpha) \, d\alpha \right] \cdot \left[\int f_j(\alpha') \, W(\alpha') \, d\alpha' \right]. \qquad (8)$$

Indeed, it is in this last sense that "error independence" is introduced as a necessary hypothesis in most works on LHV [5]. Now, if we want to include some technologically controlled element, an additional dependence in the form of a "detection efficiency", what we have to do is to redefine the overall detection probabilities as

$$P_i^{(exp)}(det) \equiv \mu \cdot P_i^{(W)}(det), \qquad (9)$$

$$P_{i,j}^{(exp)}(det) \equiv \mu^2 \cdot P_{i,j}^{(W)}(det), \qquad (10)$$

where $0 \leq \mu \leq 1$ plays the role of such efficiency, and where we are obviously implementing the hypothesis of "error independence": this time not only "on average" but also for each particular realization of α, i.e., we also have

$$P_i^{(exp)}(det|\alpha) \equiv \mu \cdot P_i^{(W)}(det|\alpha), \qquad (11)$$

$$P_{i,j}^{(exp)}(det|\alpha) \equiv \mu^2 \cdot P_{i,j}^{(W)}(det|\alpha). \qquad (12)$$

For a Bell experiment, nevertheless, it is only the average probabilities (integrated in α) that present interest, because they are the ones used to evaluate the overall result of the test. Such probabilities are estimated from the number of counts registered on a certain time-window ΔT (sufficiently large); for instance, we can estimate

$$P_i^{(exp)}(det) \approx \frac{n. \ joint \ det. \ (i,j) \ in \ \Delta T}{n. \ marg. \ det. \ (j) \ in \ \Delta T}, \qquad (13)$$

and finally we should have, assuming, as we have seen, independence of errors "on average":

$$P_{i,j}^{(exp)}(det) \approx P_i^{(exp)}(det) \cdot P_j^{(exp)}(det). \qquad (14)$$

At this point, it is time to recall that a violation of a Bell inequality only happens when the marginal detection probability (let us for simplicity assume that is equal for all detectors) surpasses a certain threshold, customarily known as "critical efficiency" (which at least as far as this framework is concerned, is a clearly misleading name: it should be substituted by, for instance, that of "critical detection probability").

However, the Wigner-PDC picture can be formulated, as proven in [7], as a local theory, therefore abiding to all possible Bell inequalities, even for $\mu = 1$. For other values of μ, marginal probabilities cannot be increased (nor can the joint ones), so therefore there is no physical way of externally raising those "efficiencies" so as to violate the inequality. In other words, unless the events of "failed detection" and "absence of emission" can be somehow distinguished (which seems difficult), the customary game of Local Hidden Variable (LHV) models and critical efficiencies, well defined as it may be, has never found (to my knowledge) an interpretation as realistic as the one we obtain with the Wigner-PDC picture that we advocate here: critical efficiencies are simply bounds on the detection rates that we can physically obtain.

Just as a proposal, we think it is not at this point unjustified to suggest the abandonment of the usual (photon, particle-based) Hilbert space description of all (PDC-generated) light states, in favour of an also quantum, but field-based description (QED), a description that finds a one-to-one equivalent in a picture of continuous, random variables (the Wigner-PDC picture), compatible with local realism and that gives rise, as natural features, to (detection probability) *enhancement* and (detection) *inefficiency*, this last another misleading term: we should perhaps use something like "absence of detection" or simply "reduced detection probability". While the first of those two phenomena is simply ignored in the customary description, the second is introduced as a mere external postulate to match observations.

Last comments: all we are doing in this paper is *comparing* two different models; of course, other more refined ones can also be build, introducing non-punctual detectors and many other features. All this modifications are desirable but not of our interest here: we just wanted to show that there is a basic feature (reduced detection efficiency) arising naturally from the mathematical structure of one of them, while it is absent in the other... a feature that seems to fit well with the experimental evidence (or absence of it, given the elusive character, after several decades, of a conclusive proof of the incompatibility of QM and local realism: so far, all observed violations of a Bell inequality do always involve additional hypothesis [6]).

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[1] Parametric Down Conversion (PDC): a pair of entangled photons is obtaining by pumping a laser beam into a non-linear crystal. Within the Winger picture [2], the pair generation can be interpreted as the non-linear mix of the laser frequency with the vacuum components.

[2] (i) A. Casado, T.W. Marshall and E. Santos. *J. Opt. Soc. Am. B* **14**, 494 (1997).
(ii) A. Casado, A. Fernández-Rueda, T.W. Marshall, R. Risco-Delgado, E. Santos. *Phys. Rev. A*, **55**, 3879 (1997).
(iii) A. Casado, A. Fernández-Rueda, T.W. Marshall, R. Risco-Delgado, E. Santos. *Phys. Rev. A*, **56**, 2477 (1997).
(iv) A. Casado, T.W. Marshall, E. Santos. *J. Opt. Soc. Am. B* **15**, 1572 (1998).
(v) A. Casado, A. Fernández-Rueda, T.W. Marshall, J. Martínez, R. Risco-Delgado, E. Santos. *Eur. Phys. J. D* **11**, 465 (2000),
(vi) A. Casado, T.W. Marshall, R. Risco-Delgado, E. Santos. *Eur. Phys. J. D* **13**, 109 (2001).
(vii) A. Casado, R. Risco-Delgado, E. Santos. *Z. Naturforsch.* **56a**, 178 (2001).
(viii) A. Casado, S. Guerra, J. Plácido. *J. Phys. B: At. Mol. Opt. Phys.* **41**, 045501 (2008).
(ix) A. Casado, S. Guerra, J. Plácido. *Advances in Mathematical Physics* (2010).
See also A. Casado, PHD Thesis.

[3] For the vacuum amplitudes (one mode, all relevant wavevectors and polarizations), $W(\alpha, \alpha^*)$ is a *gaussian*, therefore normalized and positively defined $\forall \alpha$.
It is also true that for some other states different than the vaccum state, W is not always positive: we will treat this

issue in future works.

[4] Enough example should be that the Franson experiment (at least in its original setup) is perfectly (locally and wave-likely) explained in this formalism, see one of the papers, perhaps (ii), in [2], more detail in A. Casado's, PHD Thesis.

[5] For reference on how, and under which hypothesis LHV models are built:

(i) A. Cabello, D. Rodríguez, I. Villanueva. Phys. Rev. Lett. **101**, 120402 (2008),

(ii) A. Cabello, J. -Å. Larsson, D. Rodríguez. Phys. Rev. A **79**, 062109 (2009),

(iii) D. Rodríguez, "Detection probability enhancement as a natural feature of Local Hidden Variable models". ArXiv.

[6] The two usual ones are *fair sampling* (see Clauser-Horne-Shimony-Holt's 1969 famous paper) and *no-enhancement* (see Clauser-Horne's 1974 paper).

[7] D. Rodríguez, "Wigner-PDC description of photon entanglement can still be made completely local". ArXiv.

[8] D. Rodríguez, "Revisiting factorability and indeterminism". ArXiv.

[9] This non-factorability arises naturally in the LHV's cited in [5], when one tries to introduce the "non-detection" instructions as a purely random element, i.e., we do not distinguish between states λ predetermining or not such non-detections.

Besides, we are aware, following our recent work in [8], that non-factorability can only be the result of other hidden variables (HV), relevant for the state of the source, not appearing explictly in the expressions for the predictions of the theory: this makes perfect sense, as here we are only setting the focus on vaccum amplitude as HV's... what about the laser, the crystal?